

## Problem 2.6

[Difficulty: 1]

**2.6** A velocity field is specified as  $\vec{V} = axy\hat{i} + by^2\hat{j}$ , where  $a = 2 \text{ m}^{-1}\text{s}^{-1}$ ,  $b = -6 \text{ m}^{-1}\text{s}^{-1}$ , and the coordinates are measured in meters. Is the flow field one-, two-, or three-dimensional? Why? Calculate the velocity components at the point  $(2, \frac{1}{2})$ . Develop an equation for the streamline passing through this point. Plot several streamlines in the first quadrant including the one that passes through the point  $(2, \frac{1}{2})$ .

**Given:** Velocity field

**Find:** Whether field is 1D, 2D or 3D; Velocity components at  $(2, 1/2)$ ; Equation for streamlines; Plot

**Solution:**

The velocity field is a function of  $x$  and  $y$ . It is therefore 2D.

At point  $(2, 1/2)$ , the velocity components are

$$u = a \cdot x \cdot y = 2 \cdot \frac{1}{\text{m} \cdot \text{s}} \times 2 \cdot \text{m} \times \frac{1}{2} \cdot \text{m} \quad u = 2 \cdot \frac{\text{m}}{\text{s}}$$

$$v = b \cdot y^2 = -6 \cdot \frac{1}{\text{m} \cdot \text{s}} \times \left(\frac{1}{2} \cdot \text{m}\right)^2 \quad v = -\frac{3}{2} \cdot \frac{\text{m}}{\text{s}}$$

For streamlines

$$\frac{v}{u} = \frac{dy}{dx} = \frac{b \cdot y^2}{a \cdot x \cdot y} = \frac{b \cdot y}{a \cdot x}$$

So, separating variables

$$\frac{dy}{y} = \frac{b}{a} \cdot \frac{dx}{x}$$

Integrating

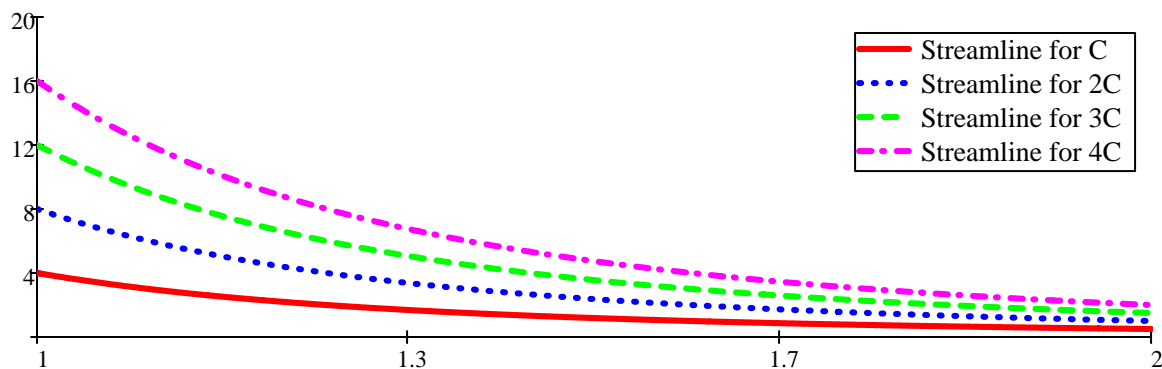
$$\ln(y) = \frac{b}{a} \cdot \ln(x) + c \quad y = C \cdot x^{\frac{b}{a}}$$

The solution is

$$y = C \cdot x^{-3}$$

The streamline passing through point  $(2, 1/2)$  is given by

$$\frac{1}{2} = C \cdot 2^{-3} \quad C = \frac{1}{2} \cdot 2^3 \quad C = 4 \quad y = \frac{4}{3x}$$



This can be plotted in *Excel*.